Euclidean Geometry Rediscovered

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The Book

Euclidean Geometry

— A Guided Inquiry Approach —

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Educational Advancement Foundation

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The Audience

- Mathematics majors
- No previous proofs-based course
- Pre-service high school math teachers
- Pre-service middle school math teachers
- Parallel graduate course for M.Ed. students.

The Dilemma

- Axiomatic Development
- Euclid
- Hilbert
- Pedagogy

Figure and Ground

"The individual perceives the environment as a total unit; he/she responds to the whole of what is seen and this whole is composed of the stimuli of which the person is aware (the figure) and those of which the person is not aware or does not attend (the ground)."

[Gestalt Center of Gainesville]

Figure and Ground

Ground

- Naïve logic and set theory
- Non-triviality
- Betweenness (for points and lines)
- Intersections (staightedge and compass)

Figure

- Congruence
- Definitions
- Constructions with straightedge and compass
- Eight axioms of plane geometry
- Theorems and proofs

Guided Inquiry
(Inquiry-Based Learning — IBL)

- The main work of the class meeting is problem-solving.
- Class goals emphasize development of skills such as communication and mathematical habits of mind.
- Most of the class time is spent on student-centered instructional activities, such as collaborative group work, student presentations, and discussion.

Guided Inquiry (IBL)

- The instructor's main role is not lecturing, but guiding, asking questions, and giving feedback; student voices predominate in the classroom.
- Students and instructor share responsibility for learning, respectful listening, and constructive critique.

Course Daily Structure

Early in course

- Brief instructor introduction of topic
- Collaborative group work
- Processing as a whole class
- Homework
- Processing

Later in course

- "May be" brief introduction of topic
- Homework
- Small group discussion of homework
- Processing as a whole class

Assessment

Grade Element	Percentage
Quizzes (weekly)	40%
Participation (daily)	20%
Notebook (6 x / semester)	20%
Final Exam	20%
Total	100%

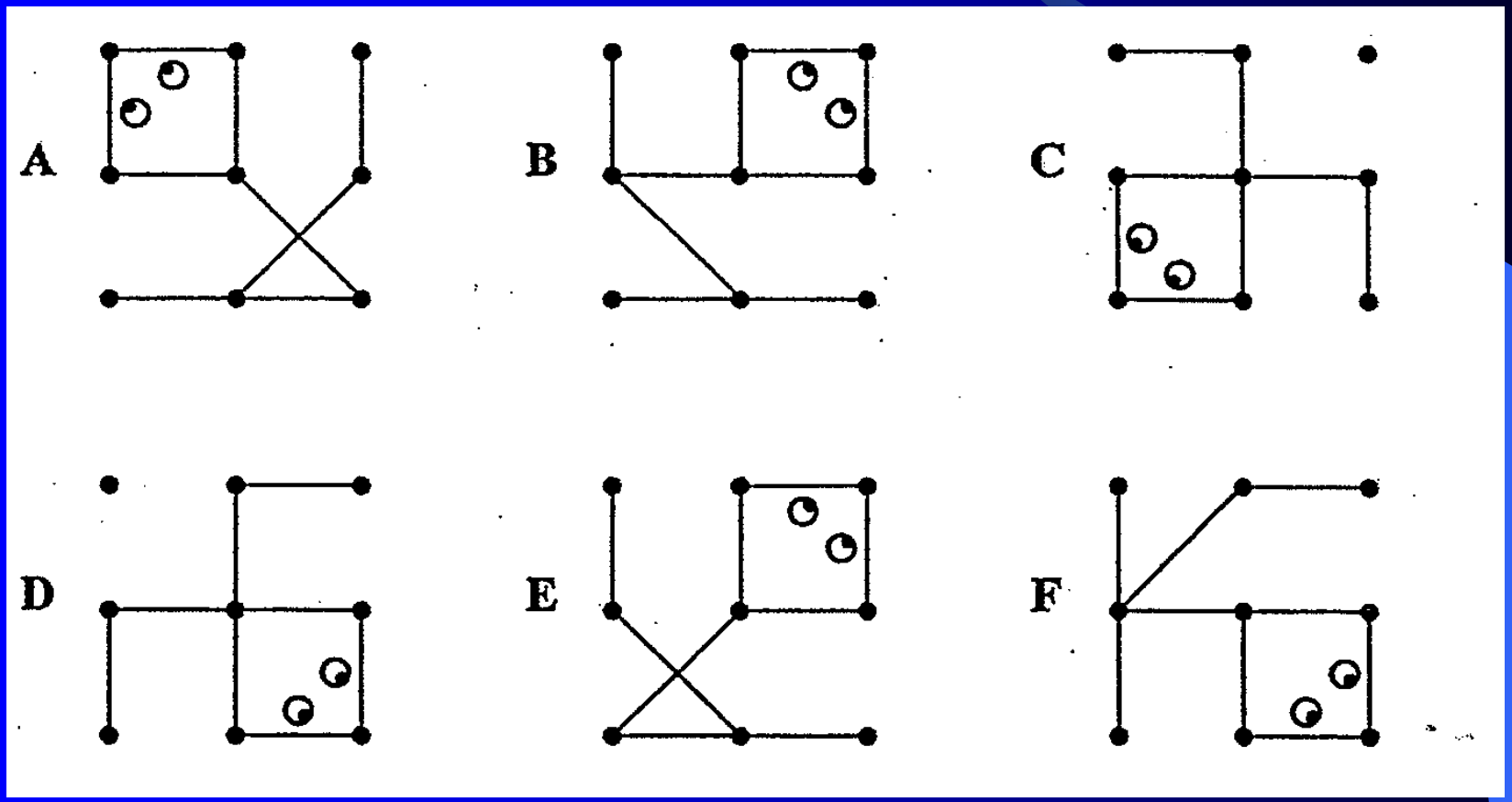
Notebook Grading

- Completeness
  6 points
- Selected Problem
  2 points
- Homework Attempt
  2 points

Grading Cycle (6)	Median Grade
N1	8
N2	8
N3	8
N4	9

Congruence and Isometries

Definition. Figure ***X*** is *congruent* to figure ***Y*** if we can orient one, or a copy of one, on top of the other so that they match exactly.



Axioms

Axiom 1: Length Measure. Each segment ***AB*** can be assigned a positive number ***L(AB)*** called the length of ***AB***, so that

i. The length of the unit segment is ***1***.

ii. Two segments are congruent if, and only if, they have the same length.

iii. If ***A***, ***B***, ***C*** are three points with ***B*** between ***A*** and ***C***, then ***L(AB) + L(BC) = L(AC).***

Axioms 2 and 3 and Justified Constructions

- Axiom 2: SSS
- Axiom 3: SAS
- Problem 22: Construct the bisector of a given angle.
- Problem 25: Construct the midpoint of a segment.

From Constructions to Theorems and Proofs

Figure ←→ Ground

- Theorem 31. Every angle has a bisector.
- Theorem 34. Every segment has a midpoint.
- Theorem 35. The base angles of an isoceles triangle are equal.
- Theorems 43 & 44: ASA and AAS.
- Theorem 45. If two angles of a triangle are congruent, then the opposite sides are congruent.

Quiz 5

- **Theorem 35A.** The line segment connecting the vertex angle of an isosceles triangle to the midpoint of the base bisects the vertex angle and is perpendicular to the base.
- Prove Theorem 35A.

Quizzes are open Book, but not open Notes.

Quiz Rubric

	Explanation: *Using pictures, symbols, and/or vocabulary to convey the path to the identified solution*	**Accuracy:** *Providing a complete and accurate solution appropriate for the given problem*
3	Explanation is clear and complete	Solution is correct and complete with no errors
2	Explanation is clear, but not quite complete.	Solution is appropriate and demonstrates understanding, but is either not quite complete or contains a minor error
1	The explanation is partially complete and/or partially developed with gaps that have to be inferred	Solution is appropriate and demonstrates some understanding, but is either not complete or contains several minor errors
0	Does not achieve minimal requirements for 1 point	Does not achieve minimal requirements for 1 point

Adapted from the Oregon Department of Education's 1995-2003 statewide assessments

Quiz Rubric – 10 points Total

	Conceptual Understanding: *Interpreting the concepts of the task and translating them into mathematics*	**Evidence Of Problem Solving:** *Choosing strategies that can work, and then carrying out the strategies chosen.*
3		
2	The translation of the task into adequate mathematical concepts using relevant information is completed	Pictures, models, diagrams, symbols, and/or words used to solve the task are complete
1	The translation of the major concepts of the task is partially completed and/or partially displayed	Pictures, models, diagrams, symbols, and/or words used to solve the task may be only partially useful and/or partially recorded.
0	Does not achieve minimal requirements for 1 point	Does not achieve minimal requirements for 1 point

From Weak to Strong

- Definition. Two lines are parallel if no point is on both lines.
- Axiom 4 (Angle Non-Congruence). If point ***B*** is in the interior of $\angle \boldsymbol{AXC}$, then $\angle \boldsymbol{AXC}$ is not congruent to $\angle \boldsymbol{AXB}$.
- Weak Alternate Interior Angle Theorem 39. If two lines have a transversal which forms alternate interior angles that are congruent, then the two lines are parallel.
  - Figure ←→ Ground

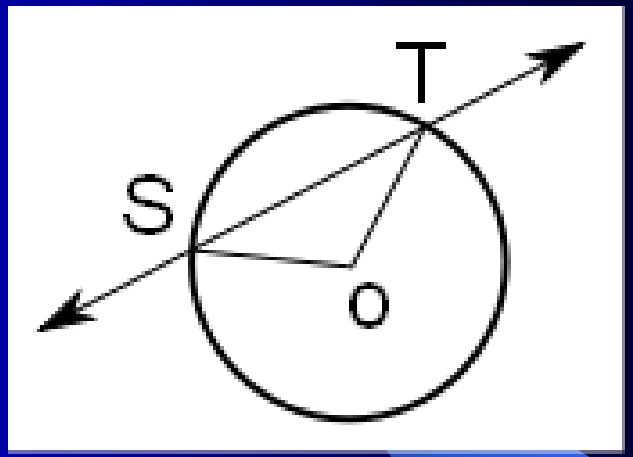
A Hard Theorem?

- Definition. A *tangent* to a circle is a line that contains exactly one point of the circle.
- Theorem 52. Let ***l*** be a line that contains a point ***T*** of circle ***c*** with center ***O***. Then ***l*** is tangent to ***c*** if, and only if, the radius ***OT*** is perpendicular to ***l***.

Quiz 10

- **Problem.** Let ***c*** be a circle with center ***O***. Let ***l*** be a line meeting ***c*** at points ***S*** and ***T*** with ***S*** not the same point as ***T***. Draw radii ***OS*** and ***OT***. Show that neither $\angle OST$ nor $\angle OTS$ can be a right angle.
- Use only theorems prior to Theorem 52.

Quiz 10



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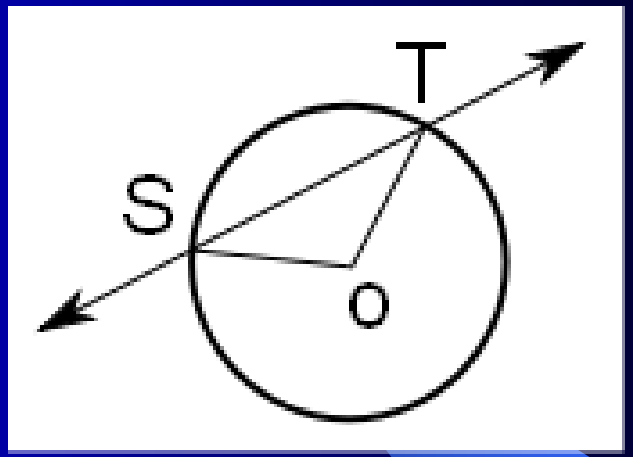
- **Theorem 46.** A triangle has at most one right angle.



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From Weak to Strong

- Axiom 5 (Parallel Lines). For every line ***l*** and every point ***P*** not on ***l***, there is at most one line containing ***P*** that is parallel to ***l***.
- Strong Alternate Interior Angle Theorem 58. Assume a transversal intersects two lines. Then the two lines are parallel if, and only if, the alternate interior angles are equal.

Quiz Scores So Far

- Quizzes are scored on a 10 point scale (by a rubric).
- 75% is lowest "B" grade for course.

Quiz	Median Grade	Quiz	Median Grade
Quiz 1	9	Quiz 6	10
Quiz 2	8	Quiz 7	8
Quiz 3	8	Quiz 8	9
Quiz 4	8	Quiz 9	9.5
Quiz 5	8	Quiz 10	10

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