Learning Through Re-Arrangement of Patterns

UNIVERSITY OF ALABAMA AT BIRMINGHAM

GREATER BIRMINGHAM MATHEMATICS PARTNERSHIP (GBMP)

JOHN C. MAYER* AND WILLIAM O. BOND

The Greater Birmingham Mathematics Partnership is funded by NSF awards EHR-0632522 and DUE-0928665.

Greater Birmingham Mathematics Partnership

Partner	Students	Minority	Reduced Lunch	MS	Gr. 6-8
Bessemer City Schools	4,087	97%	82%	1	962
Fairfield City Schools	2,323	100%	71%	1	585
Homewood City Schools	3,552	34%	22%	1	744
Hoover City Schools	11,141	22%	13%	3	2,537
Jefferson County Schools	32,553	28%	31%	7	8,713
Mt. Brook City Schools	4,150	1%	0%	1	1,016
Shelby County Schools	22,759	16%	24%	8	5,185
Trussville City Schools	4,100	8%	11%	1	970
Vestavia Hills City Schools	5,226	6%	4%	1	1,127
Univ. of Alabama at Birmingham	17,584	31%			
Birmingham-Southern College	1,412	16%			
Mathematics Education Collaborative – Bellingham, Washington					

GBMP Activities

1. Summer Courses
2. Mathematics Support Teams
3. Administrator Sessions
4. Community Mathematics Nights
5. Middle School Mathematics Teaching Certificate
6. IHE Course Development (UAB & BSC)
7. Engineering Application Tasks

GBMP Summer Courses

- *Patterns: The Foundations of Algebraic Reasoning*
  - *Also MA 313 at UAB (semester format)*
- *Patterns II: Further Explorations in Algebraic Reasoning*
- *Numerical Reasoning*
  - *Also MA 316 at UAB*
- *Geometry and Proportional Reasoning*
  - *Also MA 314 at UAB*
- *Probability*
- *Data Analysis*
- *Extending Algebraic Reasoning I and II*

Summer Courses

- Challenging nine-day mathematics content courses
- Inquiry-based
- Menu-driven
- Expandable tasks
- Multiple representations
- Manipulatives
- Collaborative group work
- Academic year follow-up sessions

Challenging Courses and Curricula

- Deepening understanding of big mathematical ideas
  - Introduce a mathematical idea by posing open-ended problems that motivate it.
- Productive disposition
  - Help students develop persistence, resourcefulness, and confidence.
- Inquiry and reflection
  - Encourage students to think critically about mathematical ideas and solutions.
- Communication
  - Value the role of communication in developing an intellectual community in the classroom.

Participant Survey

- *"This course improved my mathematical skills and understanding."*

  86% strongly agree; 12% agree

- *"The Summer course has totally changed the way I feel about myself as a user of mathematics, and therefore, my ability to help my students develop a strong understanding of mathematical concepts."*

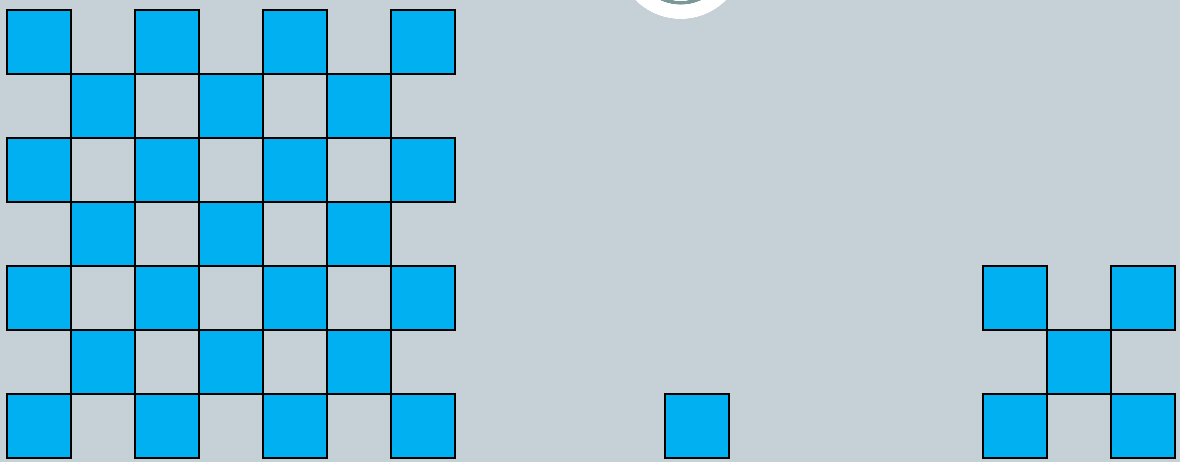
- *"I have looked closely at my questioning techniques as a result of this class. Although I have been teaching for almost 30 years, this was the first model of great questions—set in a class setting so that I could see reactions and results."*

Performance Assessment: Patterns

- MEC-developed assessment pre and post
- Scored with Oregon Department of Education Rubric: 5 + 5 + 5 + 5
- Two raters; high inter-rater reliability
- A Wilcoxon signed ranks test showed statistically significant improvement

Patterns *N* = 70	**Conceptual Understanding**		**Processes and Strategies**		**Communication**		**Accuracy**	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Median	2.0	4.0	2.0	4.0	2.0	4.0	4.0	5.0

A typical Problem: Growing Pattern C1



Stage 4 Stage 1 Stage 2

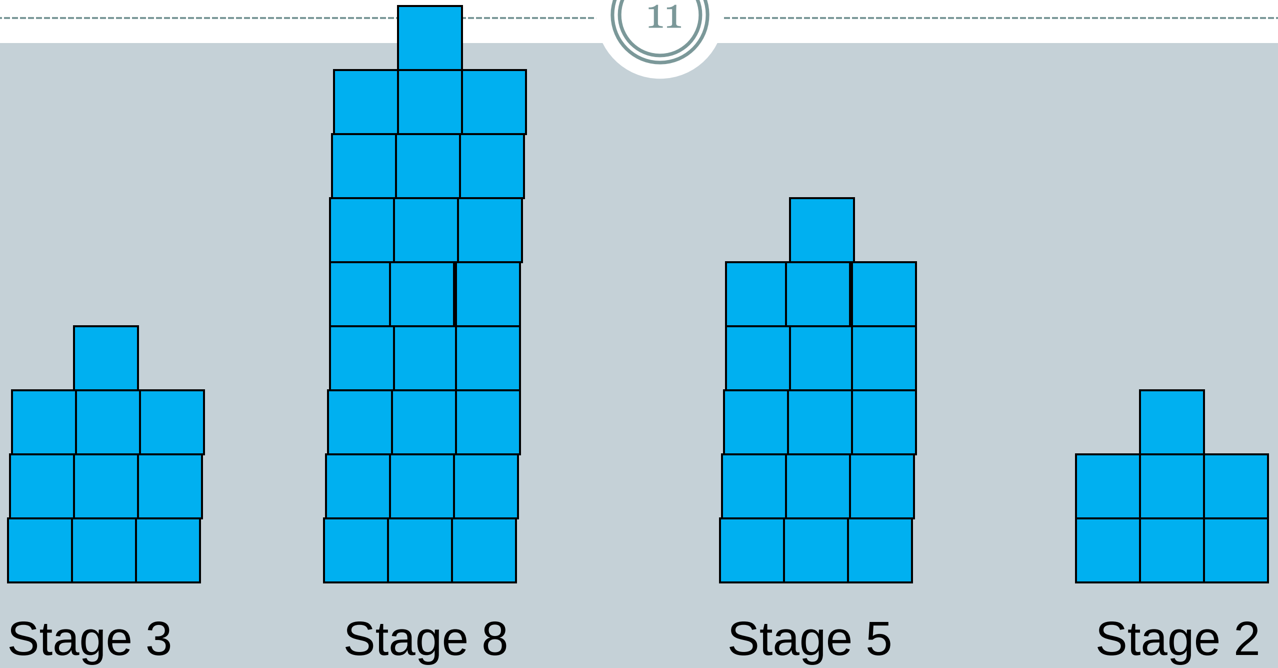
- Above are three stages in a growing pattern of square tiles.
- Build two more structures in the pattern. How many tiles will each take? How many tiles are needed for the 10^{th} structure?
- Write an algebraic rule to find the number of tiles needed for any stage of growth. Define your variables.
- Show geometrically why your rule makes sense.

Ann's Tabular Approach

Stage Number	Number of Tiles	Difference
1	1	
2	5	4
3	13	8
4	25	12
5	41	16
6	61	20
7	85	24
8	113	28
9	145	32
10	181	36

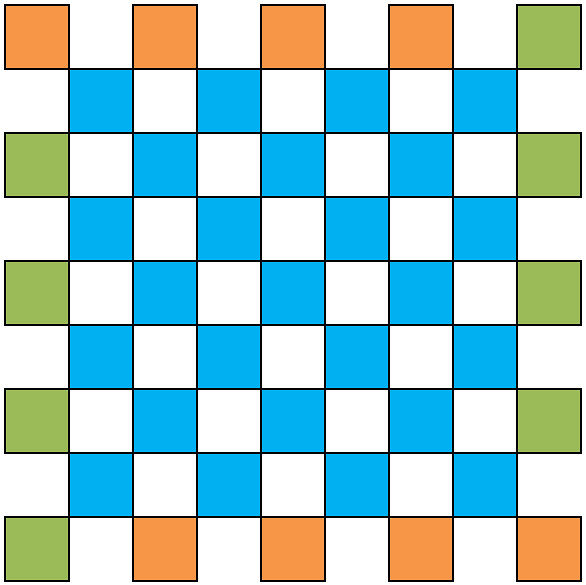
- Observation: difference increases by 4 each new stage
- Rule: *To find the number of tiles for a given stage, add a number which increases by four each time until you get to that stage.*
- Recursive understanding only
- Why did Ann do this?

Growing Pattern B1



- Above are four stages in a growing pattern of square tiles.
- Build two more structures in the pattern. How many tiles will each take? How many tiles are needed for the 100^{th} structure?
- Write an algebraic rule to find the number of tiles needed for any stage of growth. Define your variables.
- Show geometrically why your rule makes sense.

How Jack Saw Going From Stage 4 to 5



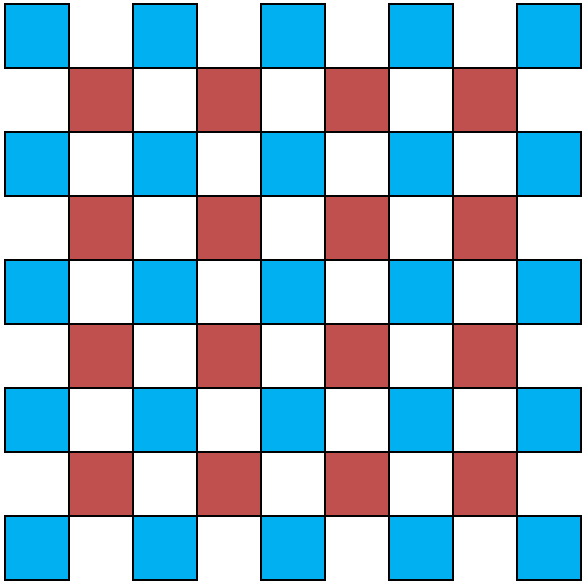
Add a layer all around going from stage 4 to stage 5.

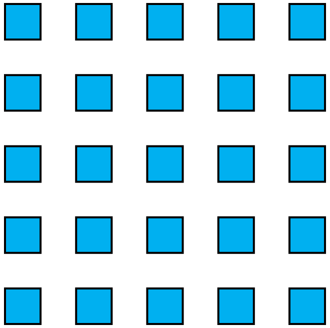
The number added is 4 times the **previous** stage number.

Shows only a recursive understanding, though expressed symbolically.

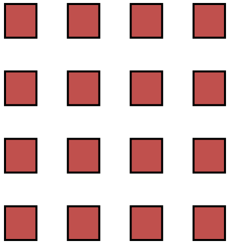
X = tiles in previous stage
n = current stage number
T = total number of tiles
Rule: $T = X + 4(n-1)$

How Ben Saw Stage 5





25 in
5 rows of 5

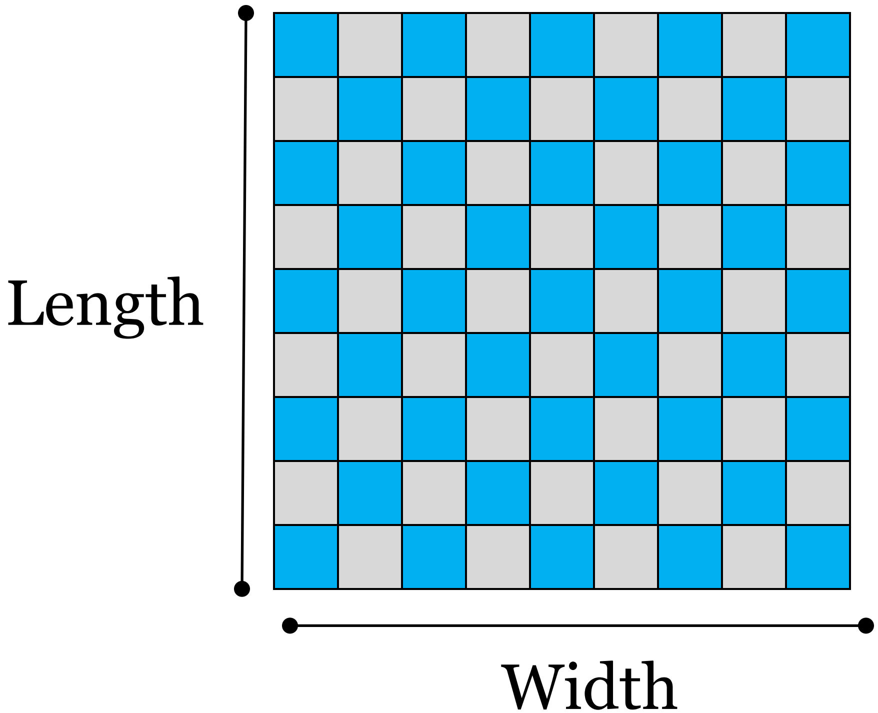


16 in
4 rows of 4

n=stage number and T=number of tiles

Algebraic Rule: $T = n^2 + (n-1)^2$

How David Saw Stage 5



Area is length times width.

Area is half blue blocks and half white blocks (almost).

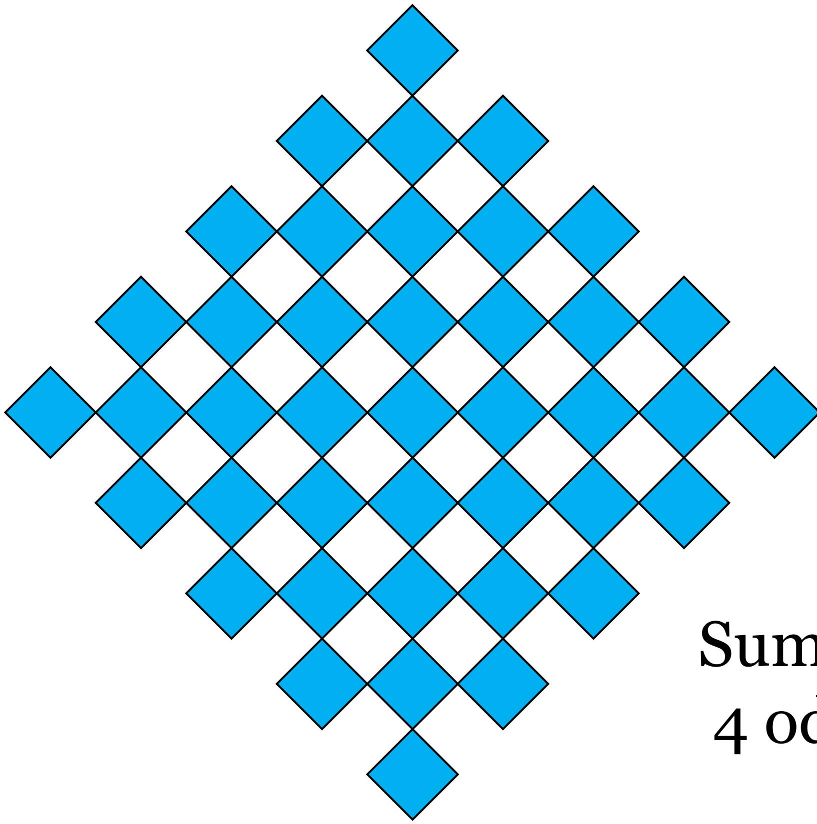
Why did David see it this way?

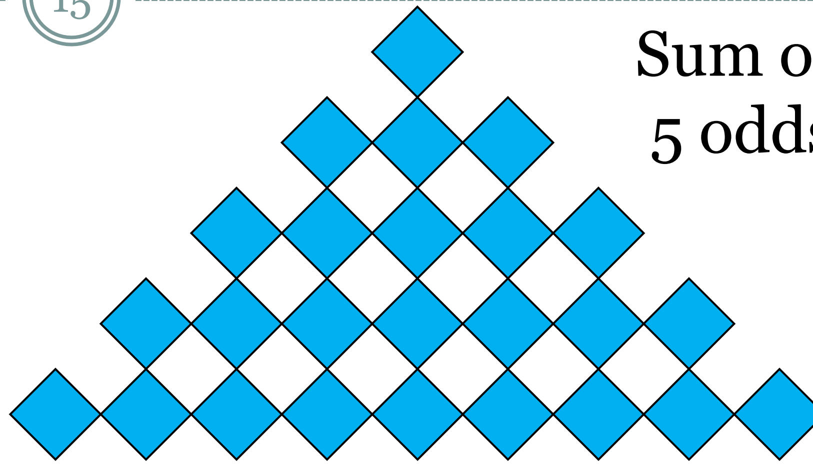
Can David make his rule more algebraic?

Rule: *Number of blue blocks is length times width, divided by 2, then rounded up.*

$$T = (2n-1)^2/2$$

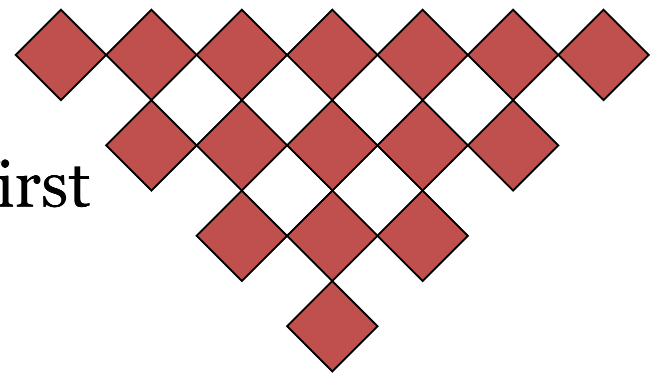
How Cary Saw Stage 5





Sum of first
5 odds

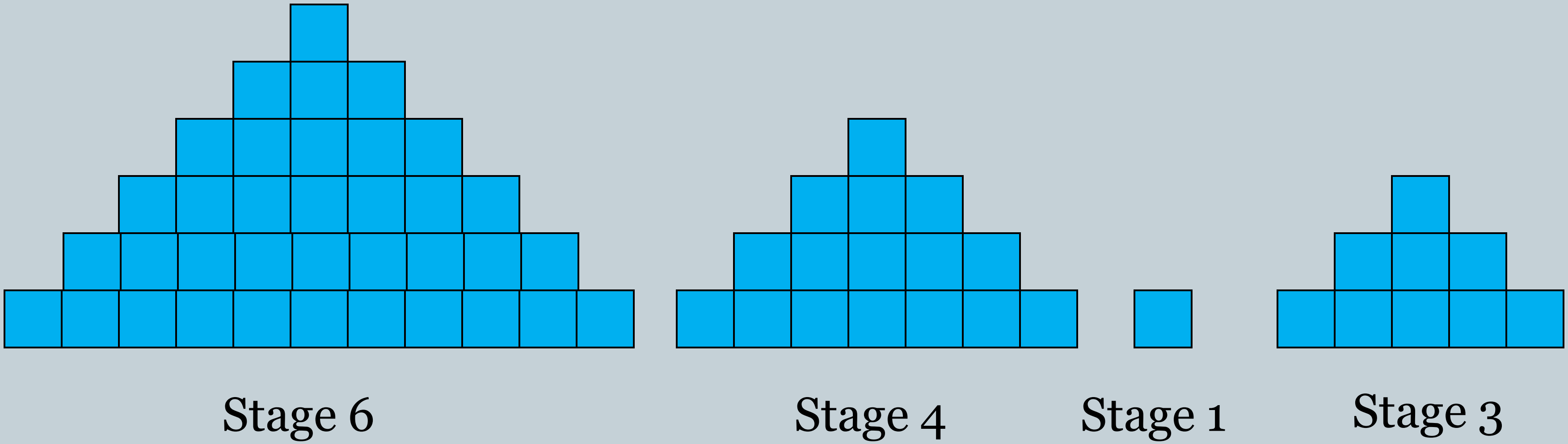
Sum of first
4 odds



Algebraic Rule: $T = n^2 + (n-1)^2$

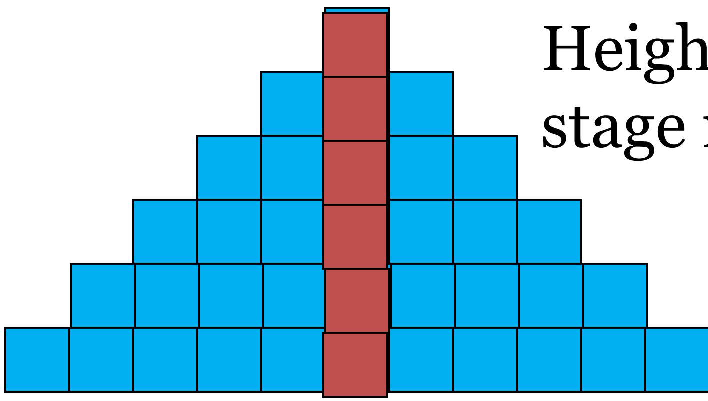
Why did Cary see it this way?

Growing Pattern A1

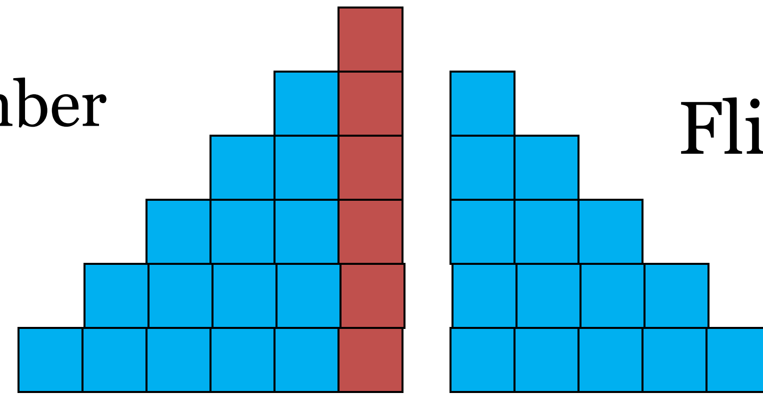


- Above are four stages in a growing pattern of square tiles.
- Build two more structures in the pattern. How many tiles will each take? How many tiles are needed for the 10th structure?
- Write an algebraic rule to find the number of tiles needed for any stage of growth. Define your variables.
- Show geometrically why your rule makes sense.

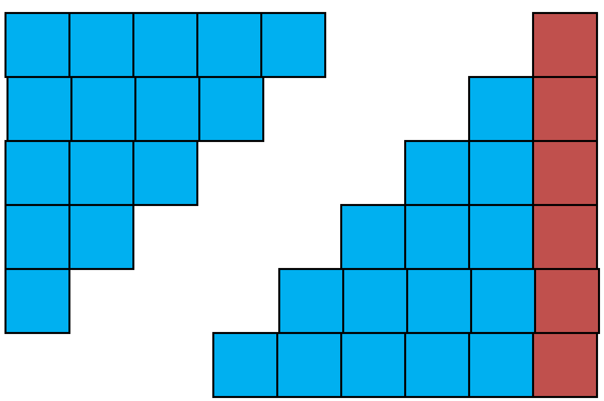
How Cary Saw Pattern A1, Stage 6

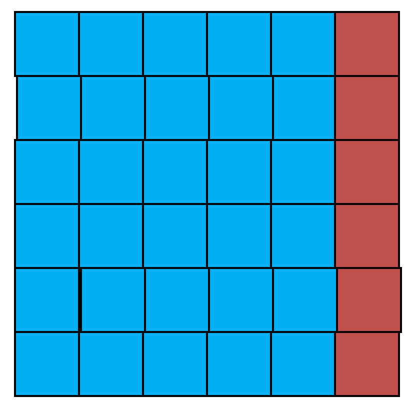


Height is
stage number



Flip!





6 by 6 square

One-Shot Manipulative Experiment

- MA 098, Basic Algebra (developmental course)
- Limited previous experience with manipulatives
- Two sections (same instructor), each split at random into two subgroups
- Treatment subgroup received Growing Pattern C1 problem with manipulatives available
- Control subgroup received Growing Pattern C1 problem **without** manipulatives available
- Collaborative group work in (random) groups of four
- Individual write-ups graded by rubric: 2 + 2 + 2 + 2 (two raters - consensus-reaching)

Statistical Results of Experiment

Rubric Item	Manipula-tives?	N	Mean	SD	Significance (2-tailed)
Conceptual Understanding	Yes	37	1.0541	0.74334	0.051
	No	35	1.4000	0.73565	
Evidence of Problem-Solving	Yes	37	1.4324	0.64724	0.352
	No	35	1.5714	0.60807	
Quality of Explanation	Yes	37	0.8919	0.87508	0.172
	No	35	0.6286	0.73106	
Accuracy	Yes	37	1.0541	0.94122	**0.006**
	No	35	1.6000	0.65079	
Total	Yes	37	4.4324	2.70330	0.184
	No	35	5.2000	2.09762	

How should we interpret these results?