Let (M^n, g) be a compact Riemannian manifold with boundary. In this lecture I will discuss the first non-zero eigenvalue problem

$$
\begin{aligned}
\Delta\varphi &= 0 \qquad \text{on} \quad M, \\
\frac{\partial\varphi}{\partial\eta} &= \nu\varphi \qquad \text{on} \quad \partial M,
\end{aligned}
\tag{1}
$$

where ν is a constant.

Problem (1) is known as the Stekloff problem because it was introduced by him in 1902, for bounded domains of the plane. In this case the problem has applications in physics. The function φ represents the steady state temperature on a domain M and the flux on the boundary is proportional to the temperature.

I will discuss the relation of this problem to harmonic analysis and some areas of differential geometry. Then I will discuss estimates of the eigenvalue ν_1 in terms of the geometry of the manifold (M^n, g). Some of the estimates I will discuss are: a sharp estimate for surfaces with non-negative Gaussian curvature, an upper estimate for a convex manifold with non-negative Ricci, a Cheger's type inquality that involves the isoperimetric constant $I(M)$ defined as

$$
I(M) = \inf_{\Omega \subset M} \frac{\mathrm{Vol}(\Sigma)}{\min\{\mathrm{Vol}(\Omega_1), \mathrm{Vol}(\Omega_2)\}},
$$

where $\Omega_1 = \Omega \cap \partial M$ is a non–empty domain with boundary in the manifold ∂M, $\Omega_2 = \partial M - \Omega_1$, and $\Sigma = \partial\Omega \cap \mathrm{int}(M)$, where $\mathrm{int}(M)$ is the interior of M.